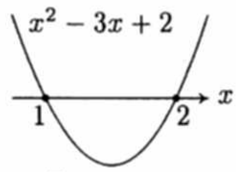
Lösungsvorschlag zu Grundaufgaben zur ln-Funktion

Nr		BE

10.1

$f(x) = \ln(x^2 - 3x + 2)$, $\quad D_f$: $x^2 - 3x + 2 = 0$: $\quad x_1 = 1$, $\quad x_2 = 2$



$$\Longrightarrow D_f = \{x \in \mathbb{R} \mid x < 1 \lor x > 2\}$$

NSt.: $\ln(x^2 - 3x + 2) = 0 \iff x^2 - 3x + 2 = 1 \iff x^2 - 3x + 1 = 0$

$$x_{1,2} = \frac{3 \pm \sqrt{9-4}}{2} = \frac{3 \pm \sqrt{5}}{2}\ , \quad x_1 = \frac{3+\sqrt{5}}{2} \approx 2{,}62\ , \quad x_2 = \frac{3-\sqrt{5}}{2} \approx 0{,}38$$

$\lim\limits_{x\to\infty} \ln(x^2 - 3x + 2) = \ln(\text{„}+\infty\text{"}) = +\infty$; $\quad \lim\limits_{x\to-\infty} \ln(x^2 - 3x + 2) = \ln(\text{„}+\infty\text{"}) = +\infty$

$\lim\limits_{\substack{x\to1\\x<1}} \ln(x^2 - 3x + 2) = \text{„}\ln(+0)\text{"} = -\infty$; $\quad \lim\limits_{\substack{x\to2\\x>2}} \ln(x^2 - 3x + 2) = \text{„}\ln(+0)\text{"} = -\infty$

$\Longrightarrow x = 1$ und $x = 2$ vert. Asymptoten von G_f

10.2

$$f'(x) = \frac{2x-3}{x^2 - 3x + 2}$$

$$f''(x) = \frac{(x^2 - 3x + 2)\cdot 2 - (2x-3)(2x-3)}{(x^2 - 3x + 2)^2} = \frac{2x^2 - 6x + 4 - (4x^2 - 12x + 9)}{(x^2 - 3x + 2)^2} =$$

$$= \frac{-2x^2 + 6x - 5}{(x^2 - 3x + 2)^2}$$

10.3

Monotonie: $\quad f'(x) = 0$: $\quad x = 1{,}5 \notin D_f$, $\quad$ keine Randpunkte $\Longrightarrow$ keine Extrema

	$x<1$	$1<x<1{,}5$	$1{,}5<x<2$	$x>2$
D_f:		$\notin D_f$	$\notin D_f$	
$2x - 3$:	$-$	$-$	$+$	$+$
$x^2 - 3x + 2$:	$+$	$-$	$-$	$+$
$f'(x)$:	$-$			$+$

f streng mon. abnehmend in $]-\infty; 1[$

f streng mon. zunehmend in $]2; \infty[$

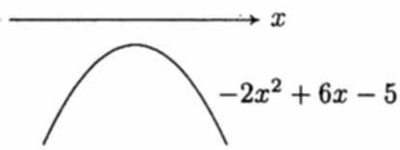
10.4

Krümmung: $\quad f''(x) = 0$: $\quad -2x^2 + 6x - 5 = 0$

Diskriminante $D = 36 - 40 < 0$

$\Longrightarrow -2x^2 + 6x - 5 < 0$ in D_f

da Nenner von $f''(x) > 0 \Longrightarrow f''(x) < 0$ in D_f



$\Longrightarrow G_f$ rechtsgekrümmt in $]-\infty; 1[$ sowie in $]2; \infty[\Longrightarrow$ kein Wendepunkt

10.5